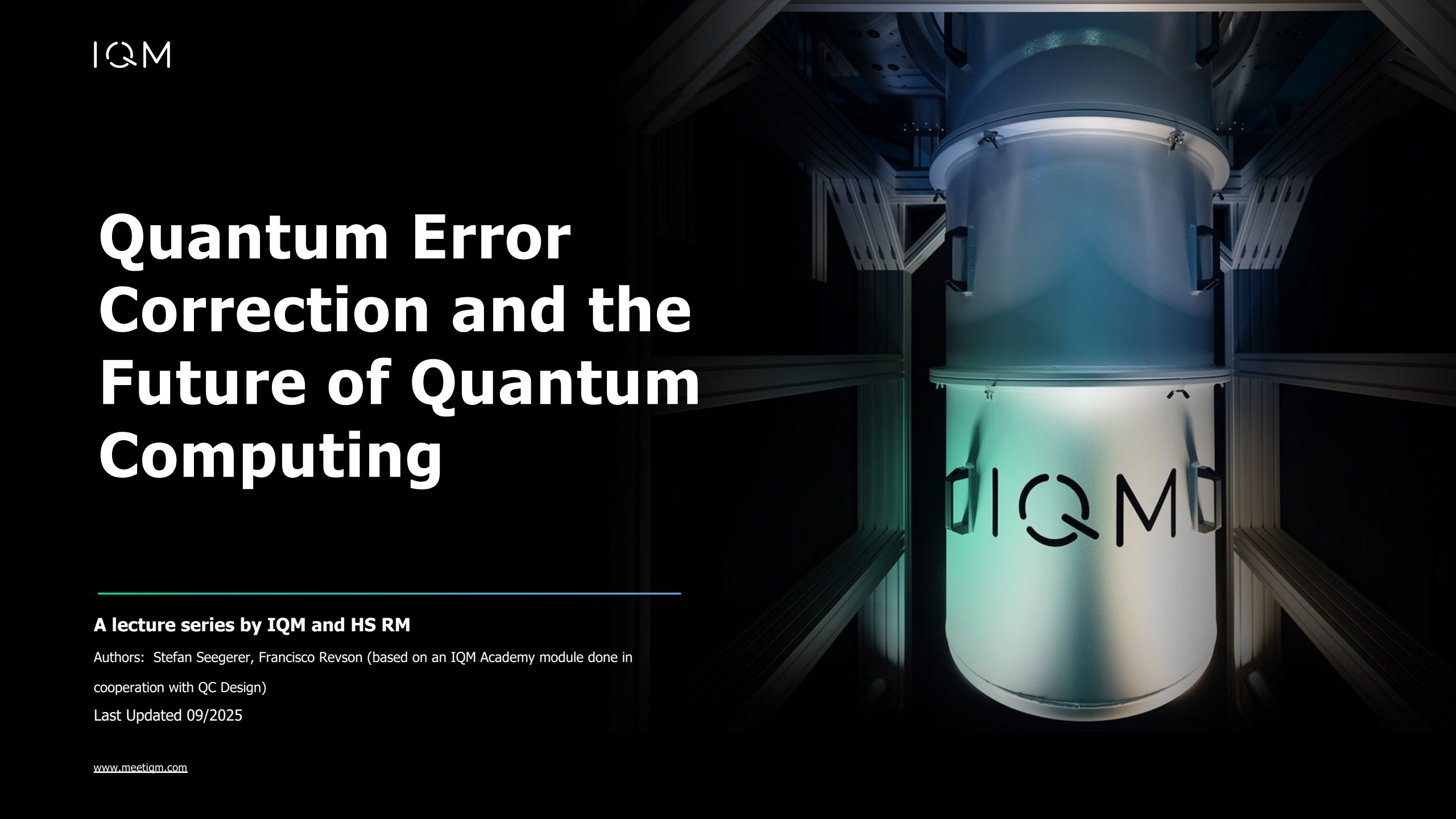
Quantum Error Correction and the Future of Quantum Computing



A lecture series by IQM and HS RM

Authors: Stefan Seegerer, Francisco Revson (based on an IQM Academy module done in cooperation with QC Design)

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Error correction in classical computing

Correcting errors in the classical world: an intuitive example



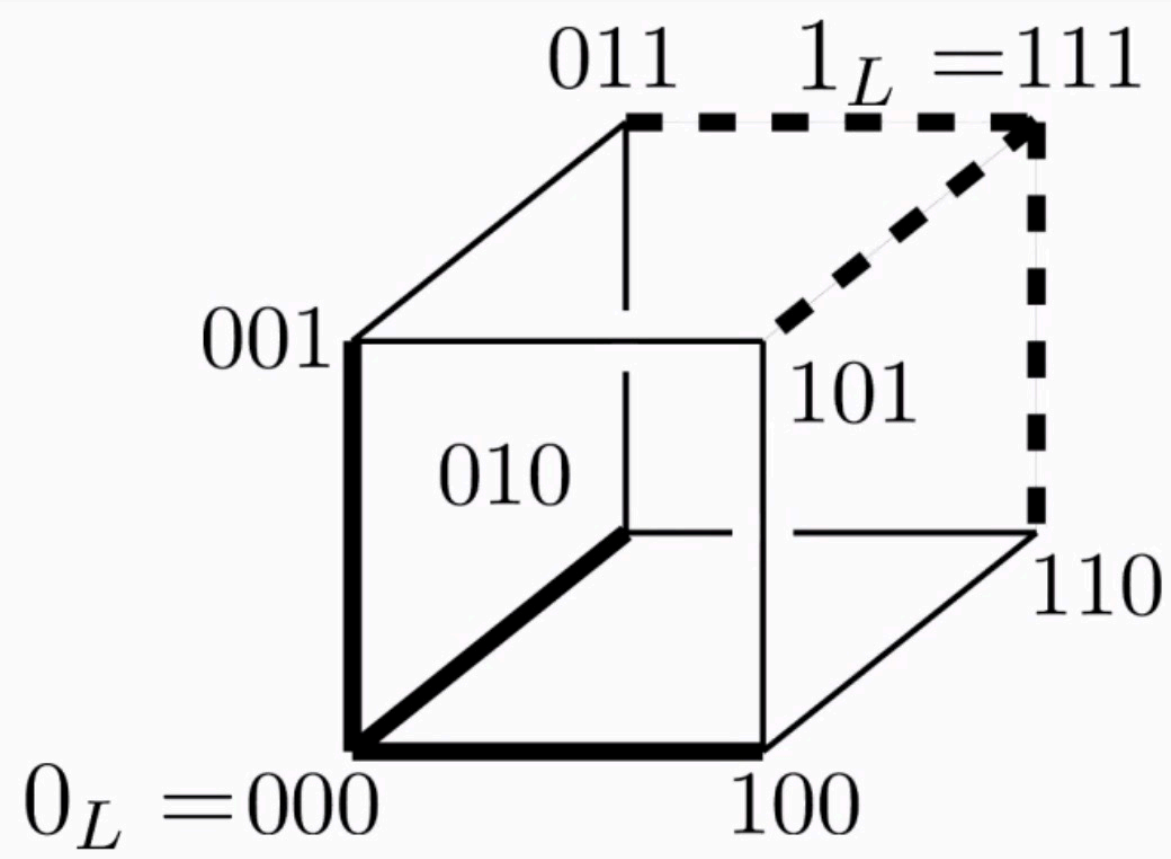
- **Alice** wants to transmit a message and there is some probability that **Bob** (the message recipient) actually gets a corrupted version of it.
- This is because **Alice** is using some noisy channel to send this message, which might introduce errors.
- **Given a certain noise level: Is there a way we can increase Bob's confidence that he received the message that Alice actually wanted to send?**

Correcting errors in the classical world: an intuitive example

- One strategy is to protect the message by encoding each bit into multiple bits.

- The process of adding extra information to a message to protect it against errors is called redundancy.

- To implement **error correction** using redundancy, Alice must define a subspace that has the same cardinality as the original (logical) space.
- Example 3-bit repetition code: Alice is moving from a space with one dimension (logical space) to a much larger space with 3 dimensions (basis elements: $\{001,010,100\}$). Both of these spaces are defined over the same set $\{0,1\}$ of order 2, whose set of basis vectors have cardinality 2 and 8, respectively.



A visualization of the 3-bit repetition code. 0_L and 1_L represent logical 0 and 1. The basis elements become apparent if we look at the axes in the picture above.

Quantum error detection

From classical error correction to quantum error correction

Making many copies of a single bit can be very useful to decrease the logical error probability classically. This, however, cannot be implemented in an analogous way in quantum mechanical systems:

- Assuming you have no knowledge of the state you're given, this cannot be copied in quantum systems because of the No Cloning theorem [Wooters, 1982].
- A quantum system cannot be measured without collapsing it to one of the eigenvector of the measured observable.
- In classical information theory the only single-bit errors are bit flips. However, in quantum information theory the single-qubit errors can be any arbitrary rotation.

Stabilizer formalism

- In quantum information, the *stabilizer formalism* [Gottesman, 1997] is used to encode k logical qubits into n physical qubits.
- This involves defining a set of operators called a *stabilizer group* $G = \{g_i\}_{i=1,\ldots,n-k}$ with $n-k$ elements.
- Each element is called a stabilizer, and they are tensor products of Pauli operators on many qubits. Pauli operators have eigenvalues ± 1, and thus a combination of Pauli operators acting on many qubits also has eigenvalues ± 1.
- We say that a state is *stabilized* by an operator g_i if the state is an eigenvector of g_i with eigenvalue $+1$.

Stabilizer formalism: Degrees of freedom

- We make use of the concept of **degrees of freedom** as a more abstract and flexible way of talking about qubits and the information contained by them. The amount of information that can be held by a system with n qubits but with only $k \leq n$ degrees of freedom is the same amount of information that can be hold by a system of k qubits with all of its degrees of freedom.
- A system with 2 qubits has, in principle, 2 degrees of freedom. The information contained by them can be represented by making use of the basis states
  - $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$
  - $\{|++\rangle, |+-\rangle, |-+\rangle, |--\rangle\}$
  - or any other arbitrary basis such as the Bell basis (normalization factor omitted):

$$\{|00\rangle + |11\rangle, |00\rangle - |11\rangle, |01\rangle + |10\rangle, |01\rangle - |10\rangle\}$$

Stabilizer formalism: Degrees of freedom

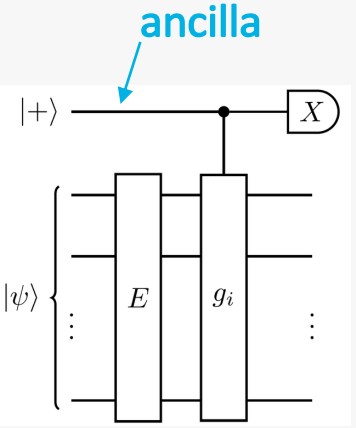
- Whenever an arbitrary quantum state is **forced to be an eigenvector of an operator**, its degrees of freedom are reduced by one.
- Example: $|\psi_1\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$ has 2 degrees of freedom.
- If we force this state to be **stabilized by the operator Z_0** (the Z operator acting on the first qubit), it means that it can only be of the form $|\psi_1\rangle = c_{00}|00\rangle + c_{01}|01\rangle$ because the Z_0 operator applied to the first qubit states leaves them unchanged, but it adds a phase of -1 to the last two states.
- The **state now only has 1 degree of freedom**, because it can represent the same amount of information as an unrestricted single qubit.
- If we make a further restriction and claim that this state is also stabilized by the operator Z_1, then the state is fully characterized by the basis state $|\psi_1\rangle = |00\rangle$ and it has no degrees of freedom.
- Given that this system must always be stabilized by the operators, **we cannot apply any operations on it without violating this restriction**, and hence this system of two physical qubits, can hold no information at all.

Stabilizer formalism: Example

- We can also say that the arbitrary state $|\psi_2\rangle$ is stabilized, not by the operators Z_0 and Z_1, but by **a single operator** X_0X_1 , which is the tensor product of two Pauli operators (but it is still only one operator!).
- In this case, the state $|\psi_2\rangle$ will be characterized by $|\psi_2\rangle = \alpha\,(|00\rangle + |11\rangle) + \beta\,(|01\rangle + |10\rangle)$
- Here, we made use of a single operator to restrict the possible states of the system, and the system now only has 1 degree of freedom and can hold the information equivalent to 1 qubit.

Syndrome of a quantum error correction code

- The stabilizers of a quantum error-correcting code can also be used to **detect the presence of errors**.
- To detect errors, one can perform non-destructive **measurements of the stabilizer operators**.
- This is achieved by using **additional ancillary qubits**, which allow one to extract the eigenvalue of the state with respect to a given stabilizer operator.



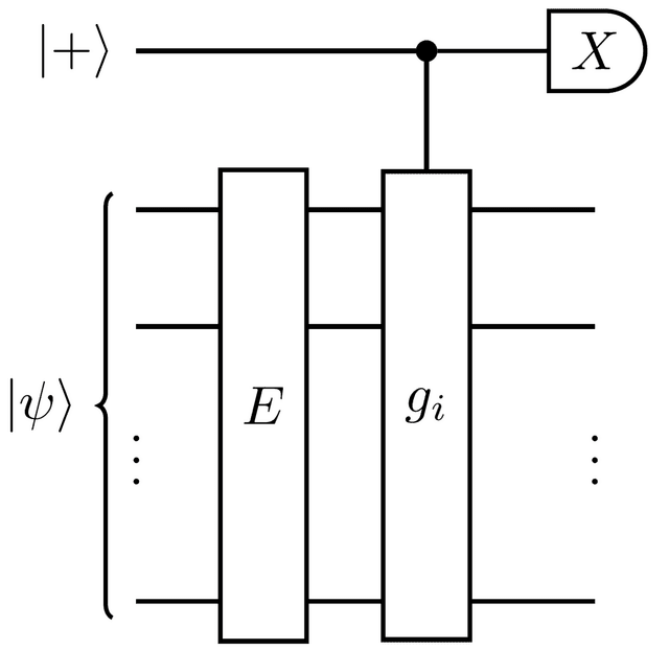
- The measurement outcome of the ancillas corresponds to the eigenvalue of the stabilizer operator, and its pattern over all stabilizers is called the syndrome of the error.

- By analyzing the syndrome, we can identify which qubits were affected by an error, but also the "type" of error, and perform corrective operations to recover the original state of the code.

Syndrome of a quantum error correction code: Example

If the stabilizer commutes with the error, the ancilla will be measured in the $|+\rangle$ state.

If the stabilizer anti-commutes with the error, the ancilla will be measured in the $|-\rangle$ state.



You can also make use of the following example, compute the evolution of the system and measure the ancilla: a single qubit is encoded in two physical qubits through the stabilizer operator X_0X_1. The codespace has the basis states $(|00\rangle + |11\rangle)$ and $(|01\rangle + |10\rangle)$, which we will call logical $|0_L\rangle$ and logical $|1_L\rangle$ respectively. We prepare our system in the state:

$$|\psi\rangle = |00\rangle + |11\rangle = |0_L\rangle$$

And we say that an error E has appeared in the code. Use the previously shown circuit to see which ancilla state you would end up with if the error is:

1. $E = Z_0$ (answer: $|-\rangle$).
2. $E = Z_1$ (answer: $|-\rangle$).
3. $E = Z_0Z_1$ (answer: $|+\rangle$).
4. $E = X_0$ (answer: $|+\rangle$).
5. $E = X_1$ (answer: $|+\rangle$).
6. $E = X_0X_1$ (answer: $|+\rangle$).

Syndrome of a quantum error correction code

- Measuring the eigenvalue of a single stabilizer operator is not enough to detect errors.
- One must perform the measurements of all stabilizers in order to obtain as much knowledge as possible regarding how the state might have been affected by the error.

Logical errors

- Logical operators commute with all the stabilizers of the code.
- This means, the measurement results won't toggle any of the stabilizers when a logical gate operation $\overline{X^{(i)}}$ or $\overline{Z^{(i)}}$ (up to a product with stabilizer operators) is performed.
- Therefore, whenever an error has the same shape as a logical operator, the error becomes undetectable. This is known as a logical error.

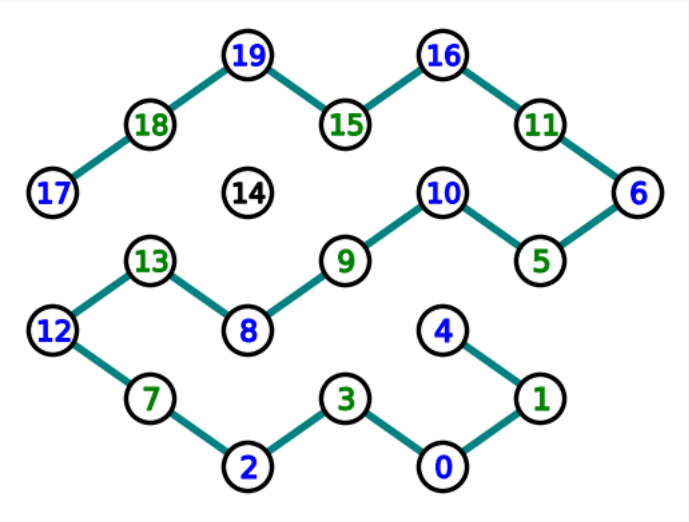
- The smallest number of errors that can result in a logical error is called the **distance** of the code.

Quantum Repetition Code

Quantum Repetition Code

- The **quantum repetition code** provides a way of choosing the stabilizers in the code.
- One can envision this code as a chain of qubits, and the stabilizers are defined by the application of Z gates on each pair of neighbouring qubits. This encoding strategy suggests that each stabilizer g_i should be chosen according to

$$g_i = Z_i Z_{i+1}$$

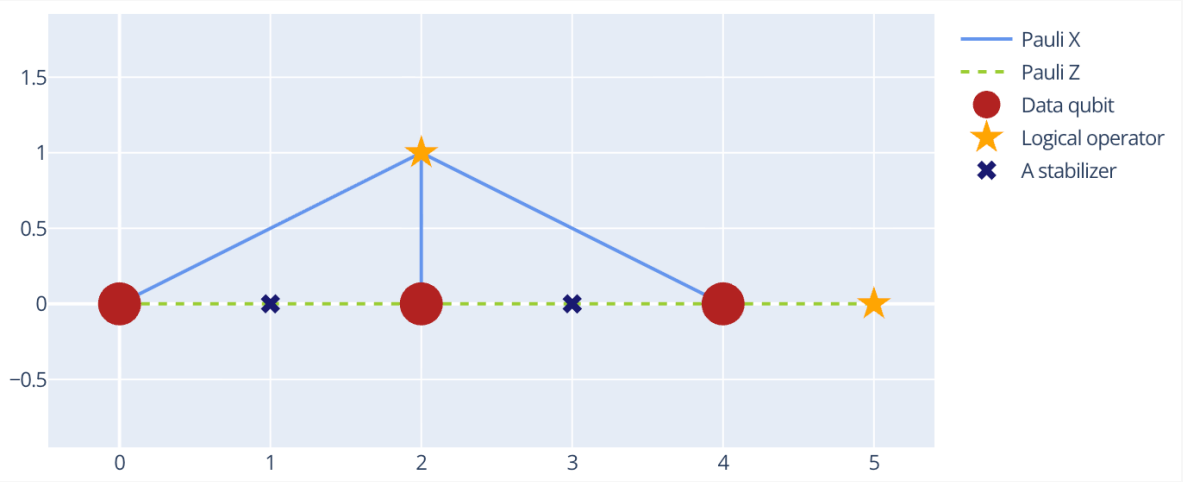


Example for chain of qubits used for repetition code experiment on IQM Garnet (more in the Lab exercises)

Quantum Repetition Code

- The repetition code encodes one logical qubit into n qubits (e.g. $|0\rangle_L = |000 \ldots 0\rangle$).

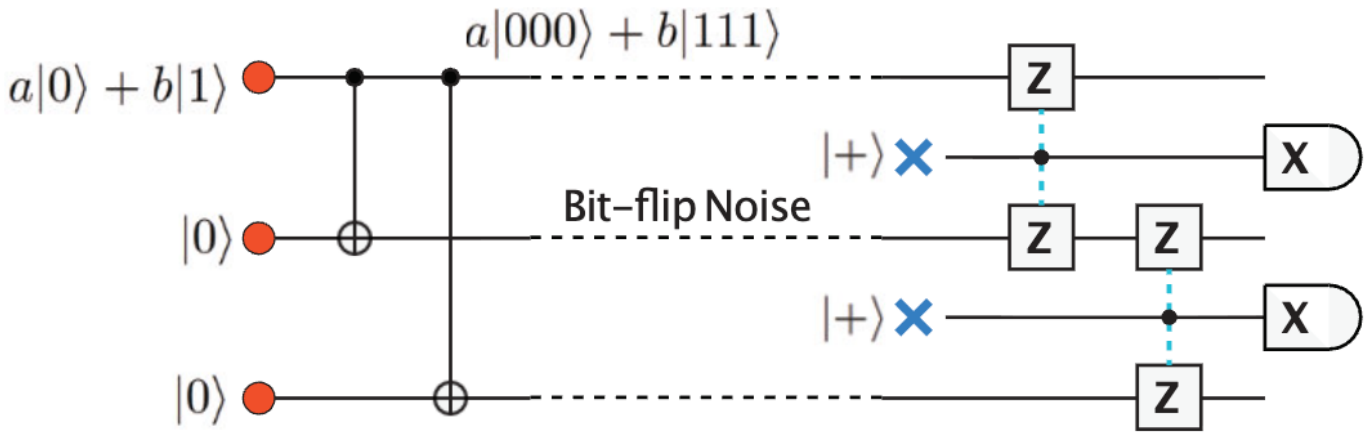
- Example:
  - 1 logical qubit is encoded in 3 data qubits
  - Stabilizer operators are Z_0Z_1 and Z_1Z_2



- The logical operators are:
  - A logical $\overline{X}$ (blue line, CNOT), obtained by applying X's on all qubits.
    The distance for this type of logical errors is equal to the number of qubits.
  - A logical $\overline{Z}$ (green dotted line, CZ), obtained by applying Z's on the last qubit of the chain.
    The distance for this type of logical errors is 1.

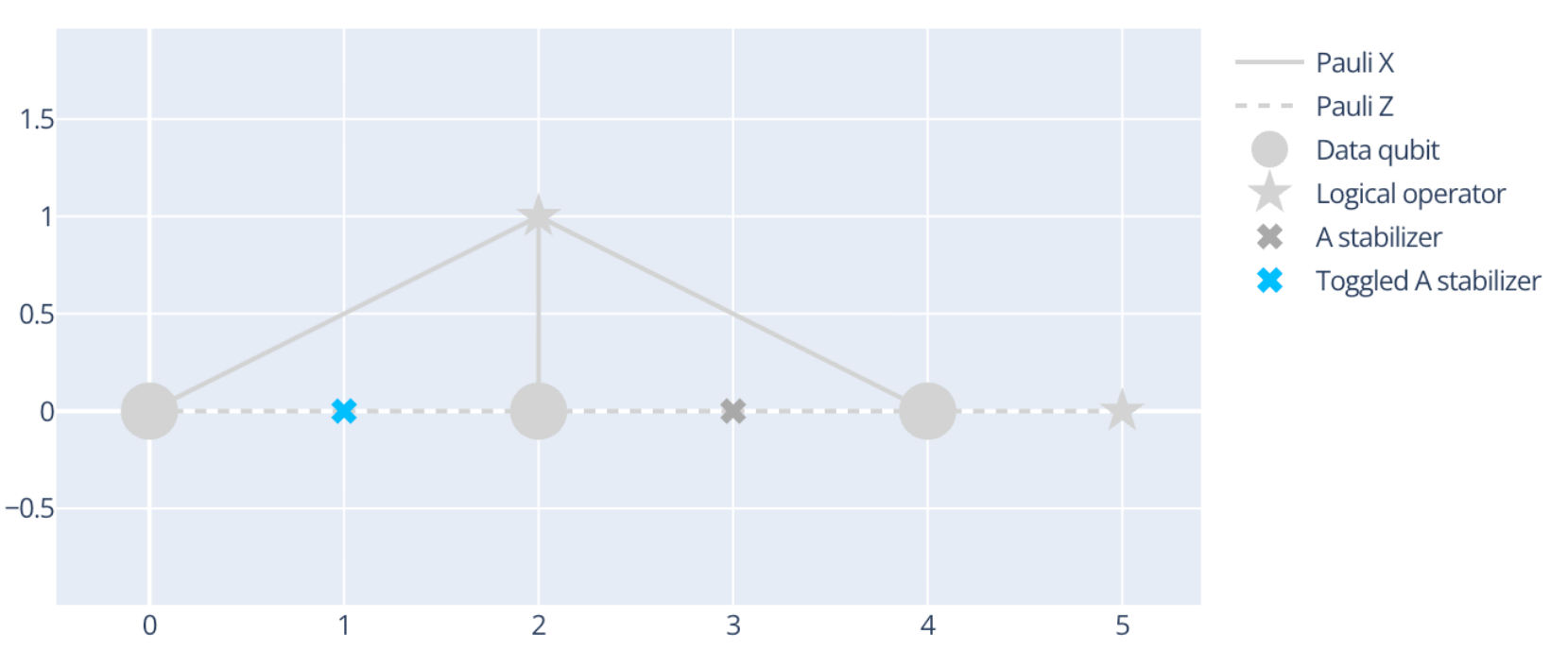
Quantum Repetition Code

If we run this on a quantum computer, we would need to express this idea as a circuit like any other quantum algorithm. The left part encodes the qubit with the repetition code. Then (after a potential bit-flip noise) two additional ancillary qubits are used to perform the stabilizer measurements.



Quantum Repetition Code

- Applying an X error on qubit 0 will toggle a stabilizer

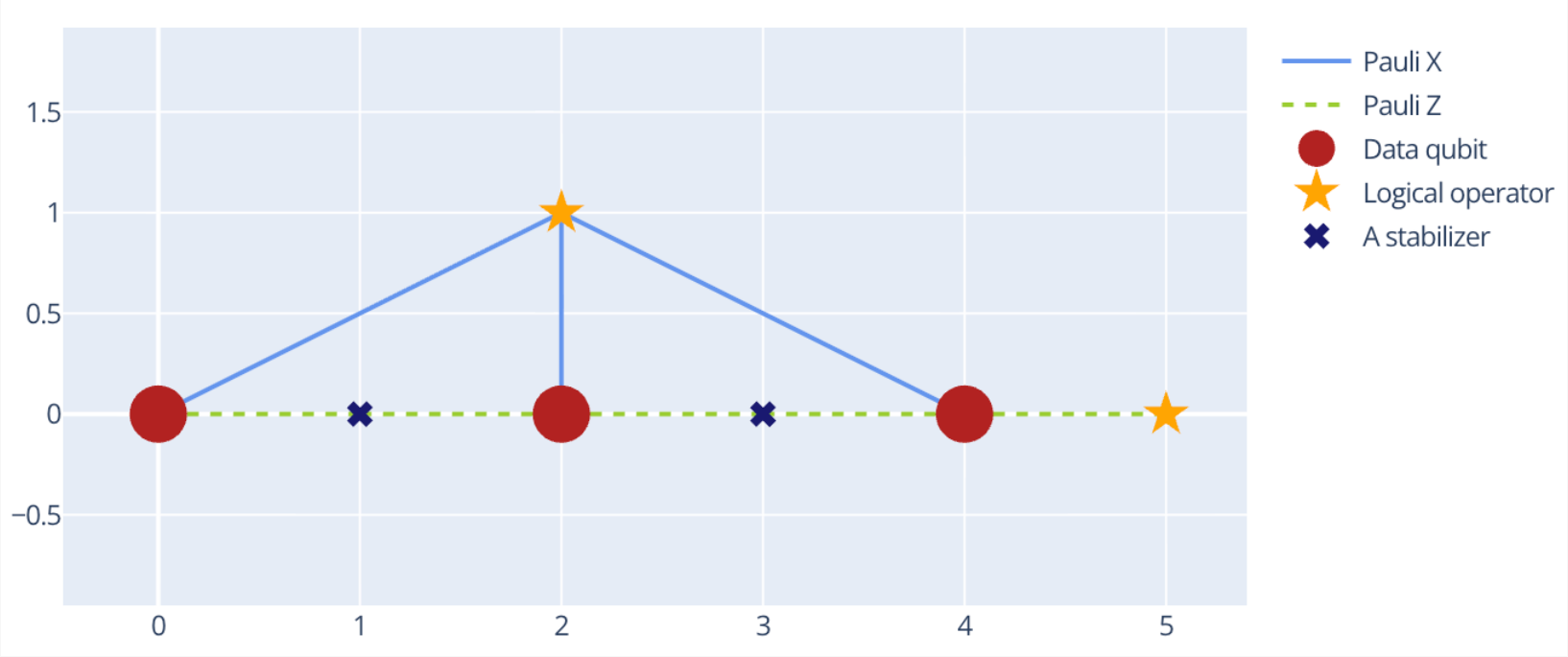


- Which stabilizer will be toggled with an X error on data qubit 1 (at (2,0))?

→ Both stabilizers

Quantum Repetition Code

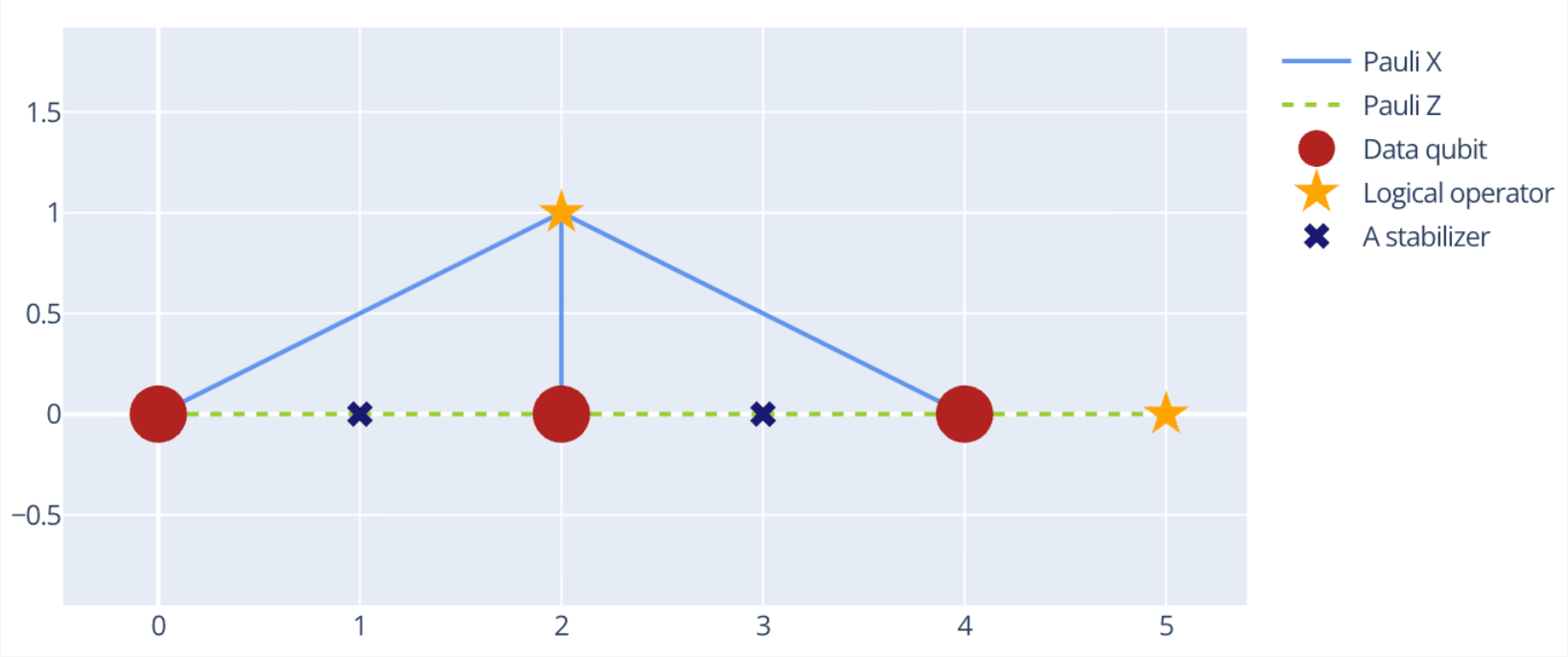
- What stabilizer will be triggered with a X error on all data qubits?



→ None, it is a logical error

Quantum Repetition Code

- What stabilizer will be triggered with a Z error on qubit 0?

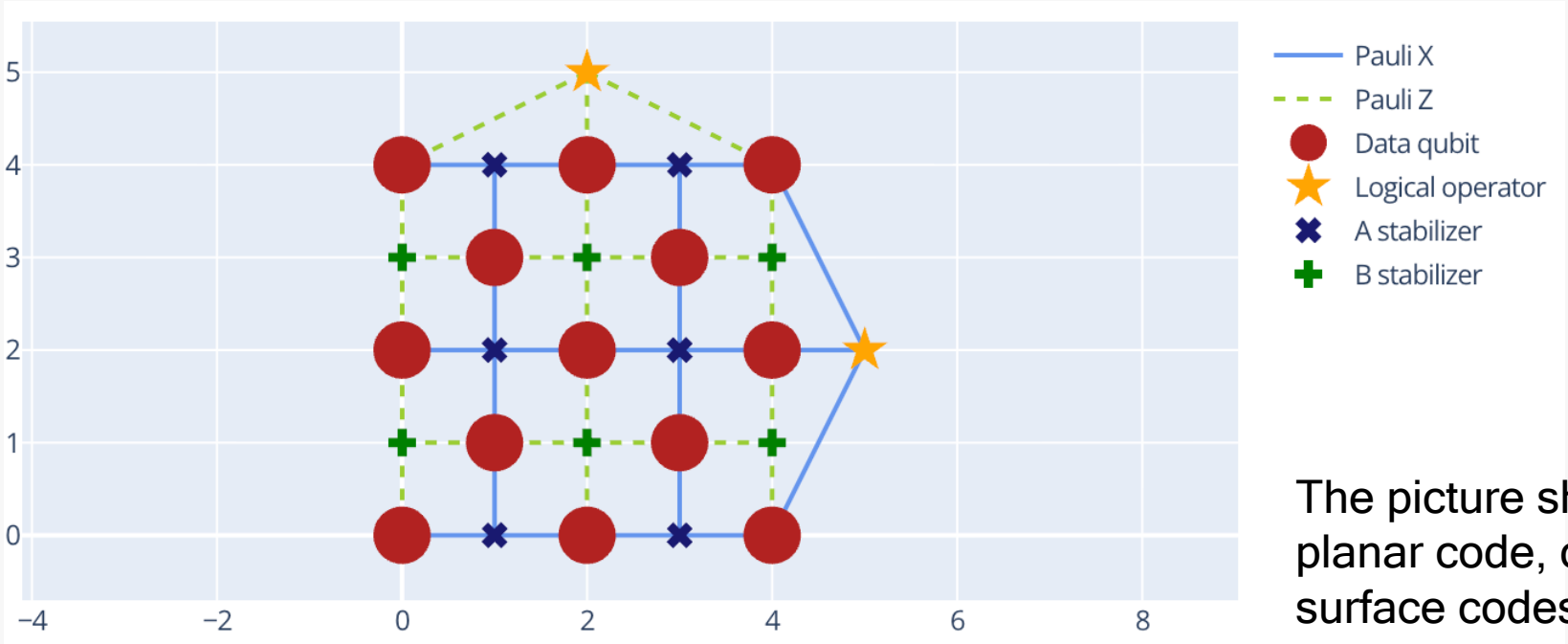


→ None, it commutes with the stabilizers

Detecting Z errors in addition to X errors with Surface Codes

Surface Codes

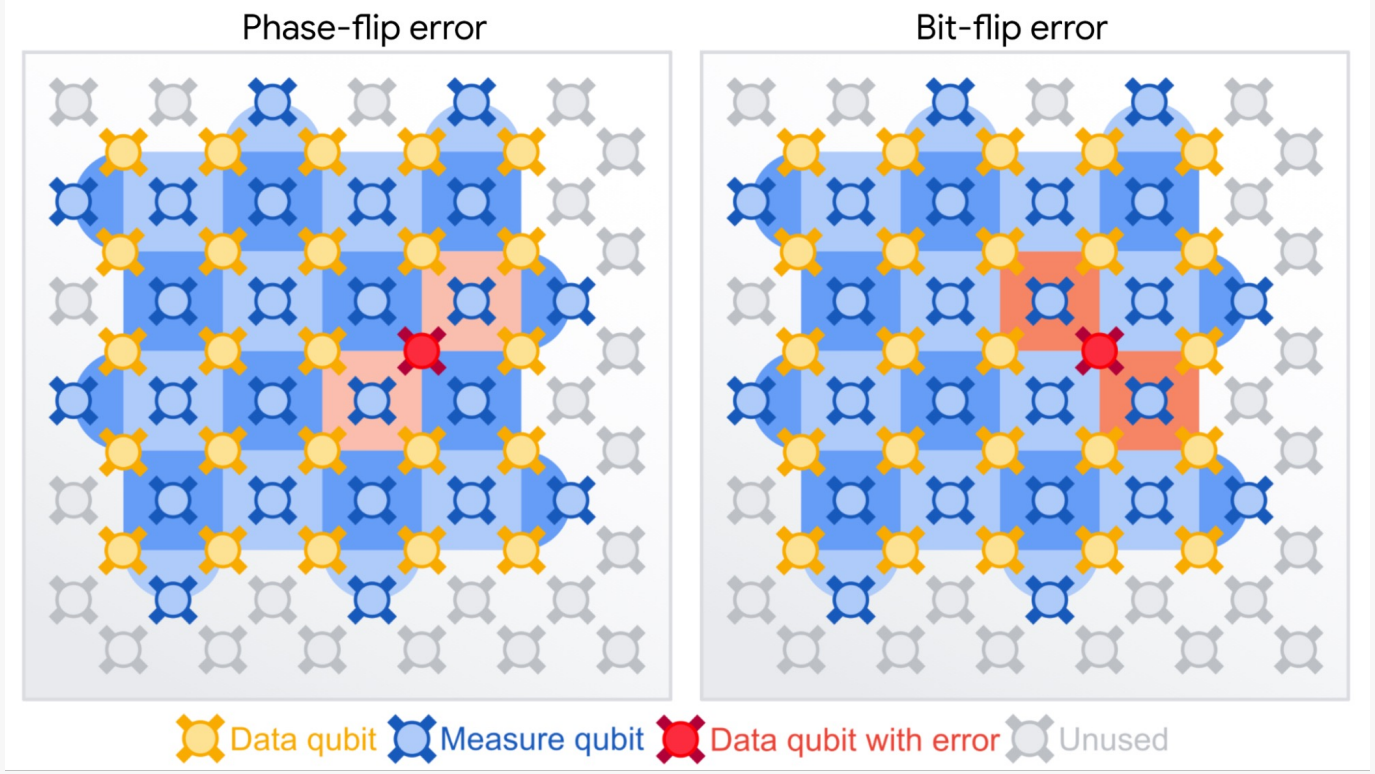
- There exists a whole class of QEC codes called surface codes. Stemming from the fact that stabilizers and logical operators are given by the distribution of qubits and ancillas in a 2D lattice [Kitaev, 1997]
- In surface codes, one defines two types of stabilizers: X-type stabilizers and Z-type stabilizers.
- This makes surface codes of great interest because they can detect for any type of error that may arise in the code, instead of only detecting errors of a single type as the quantum repetition code.



The picture shows the planar code, one of the surface codes.

Surface Code

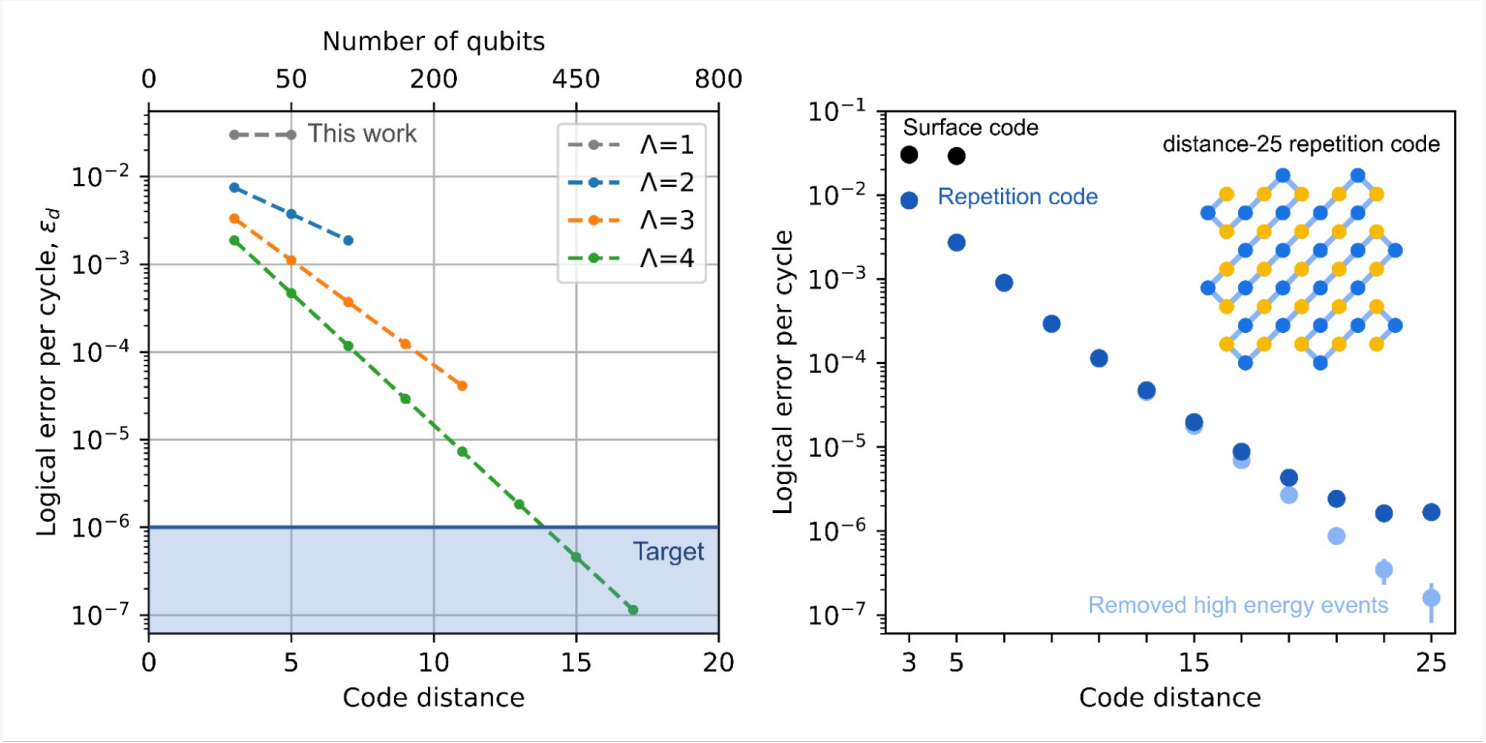
- Google (2023) showed that scaling the surface code indeed leads to a reduction in the logical error.
- Left: Expected progression as we improve performance (quantified by Λ) and scale (quantified by code distance) for surface codes. Right: Experimentally measured logical error rates per cycle versus the distance of one-dimensional repetition codes and two-dimensional surface codes.



Google Quantum AI. Suppressing quantum errors by scaling a surface code logical qubit. *Nature* **614**, 676–681 (2023). https://doi.org/10.1038/s41586-022-05434-1 (CC-BY), image taken from blog post (https://research.google/blog/suppressing-quantum-errors-by-scaling-a-surface-code-logical-qubit/)

Surface Code

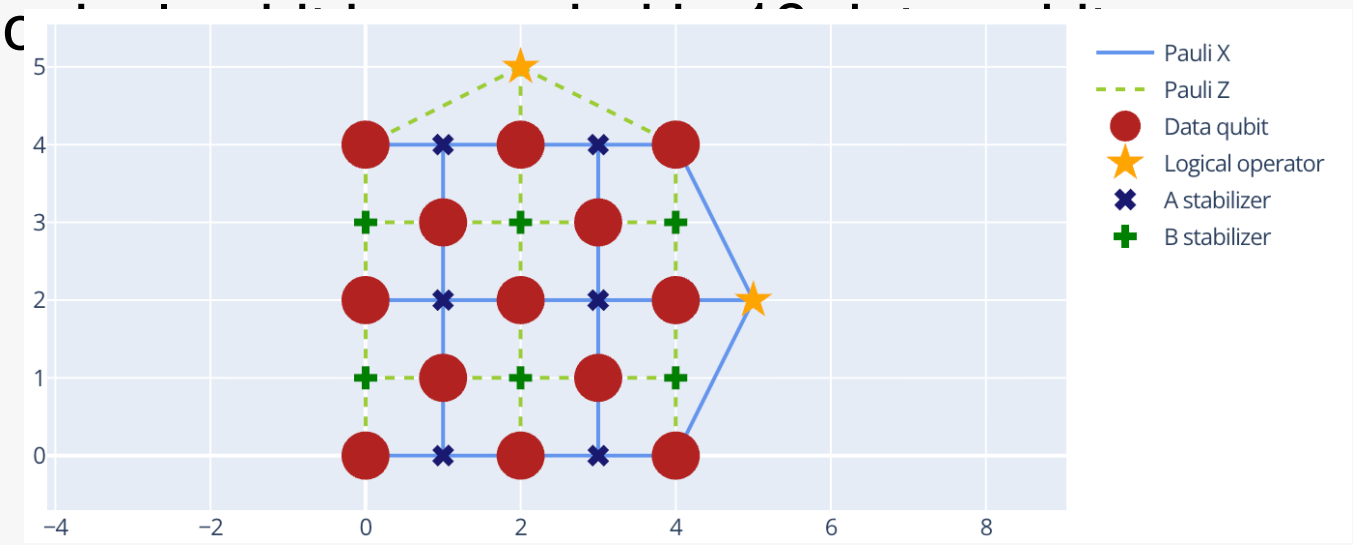
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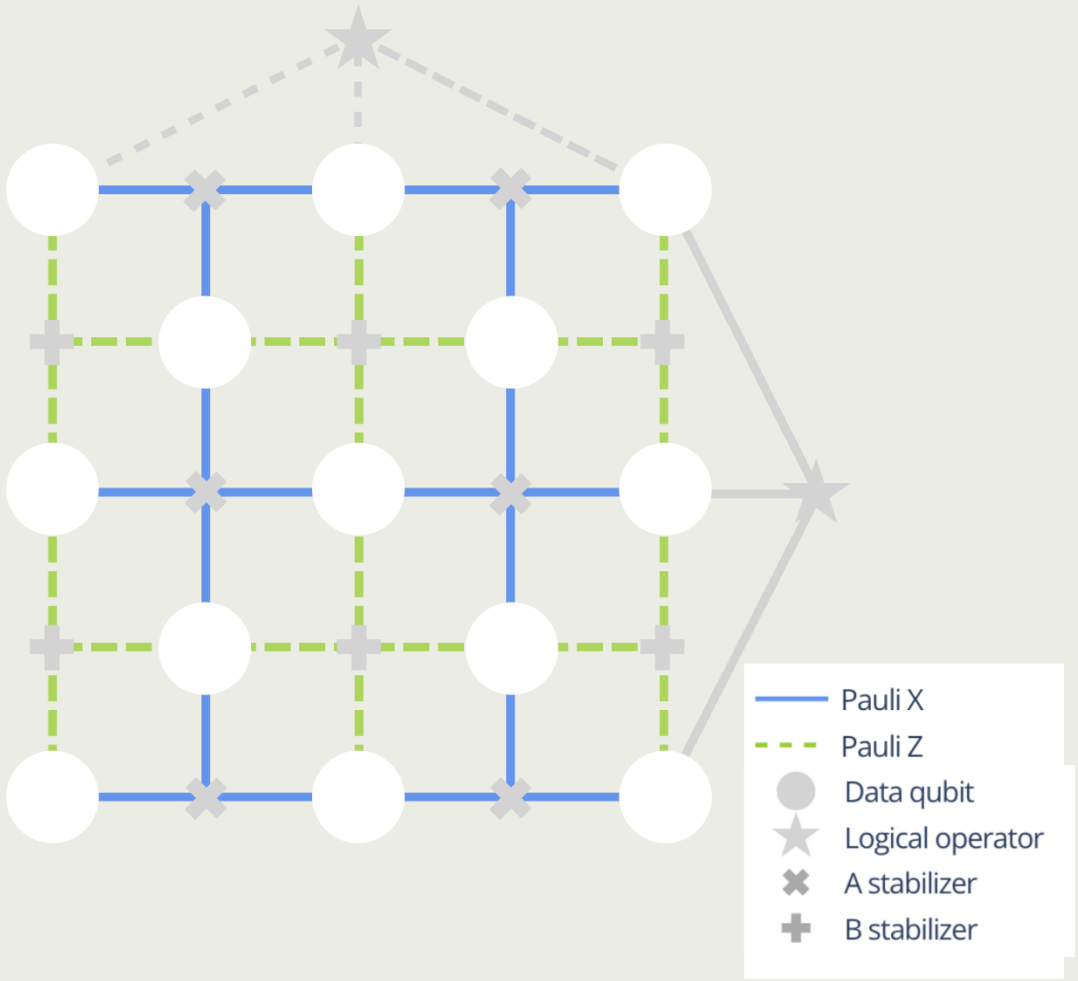
Planar Code

- The planar code with distance n encodes one logical qubit into $n^2 + (n-1)^2$ data qubits
- The logical operators are:
  - A logical $\overline{X}$ (blue line, CNOT), obtained by applying X's on all qubits on the right hand side. The distance for this type of logical errors is equal to the number of qubits (3).
  - A logical $\overline{Z}$ (green dotted line, CZ), obtained by applying Z's on the top three qubits. The distance for this type of logical errors is equal to the number of qubits (3).
- Stabilizer operators are $Z_0Z_1Z_3$ (and 5 more) as well as $X_0X_3X_5$ (and 5 more)
- Example: 1 lo



Try it yourself

- Go to https://www.iqmacademy.com/play/playQec/ and apply different errors, investigate how they can be retrieved with stabilizers.



Apply random error

Start with your qubit in state

- $|0\rangle$
- $|1\rangle$
- $|+\rangle$
- $|-\rangle$

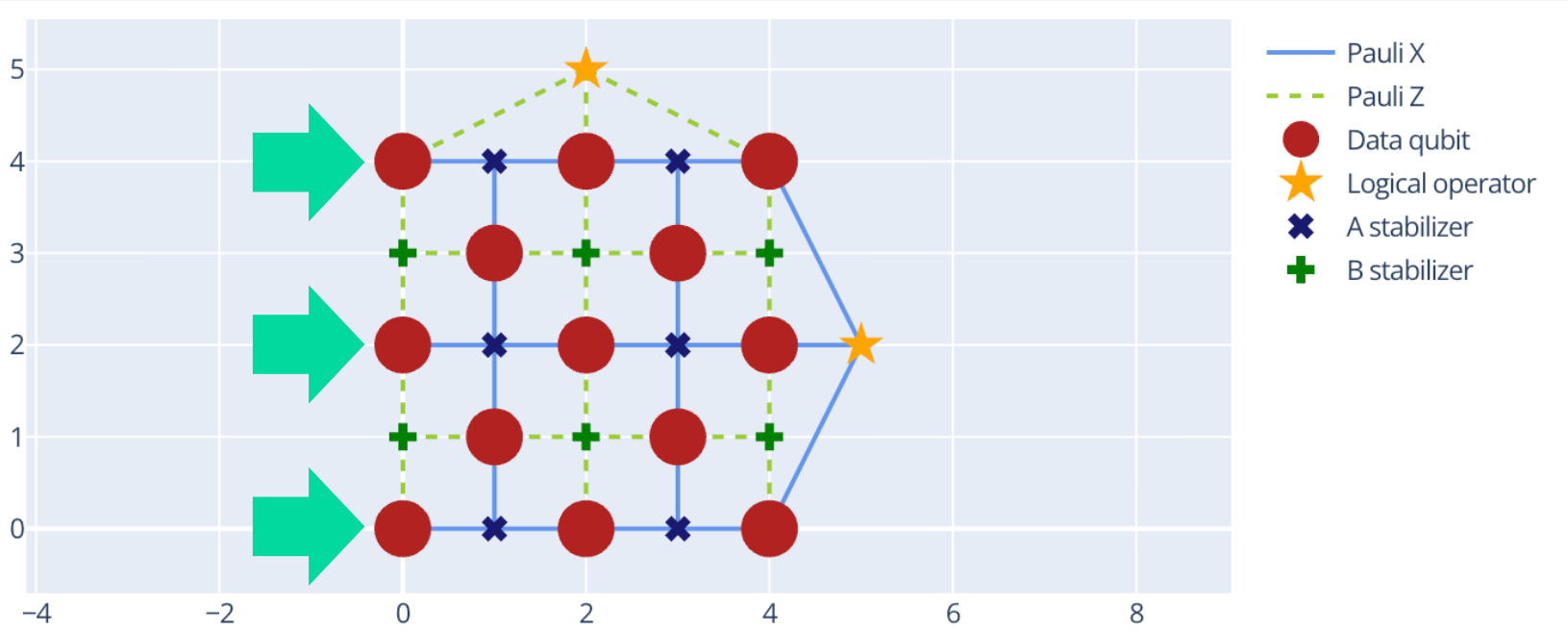
Choose what type of operation is applied on click

- X -gate
- Z -gate

IQM

Planar Code

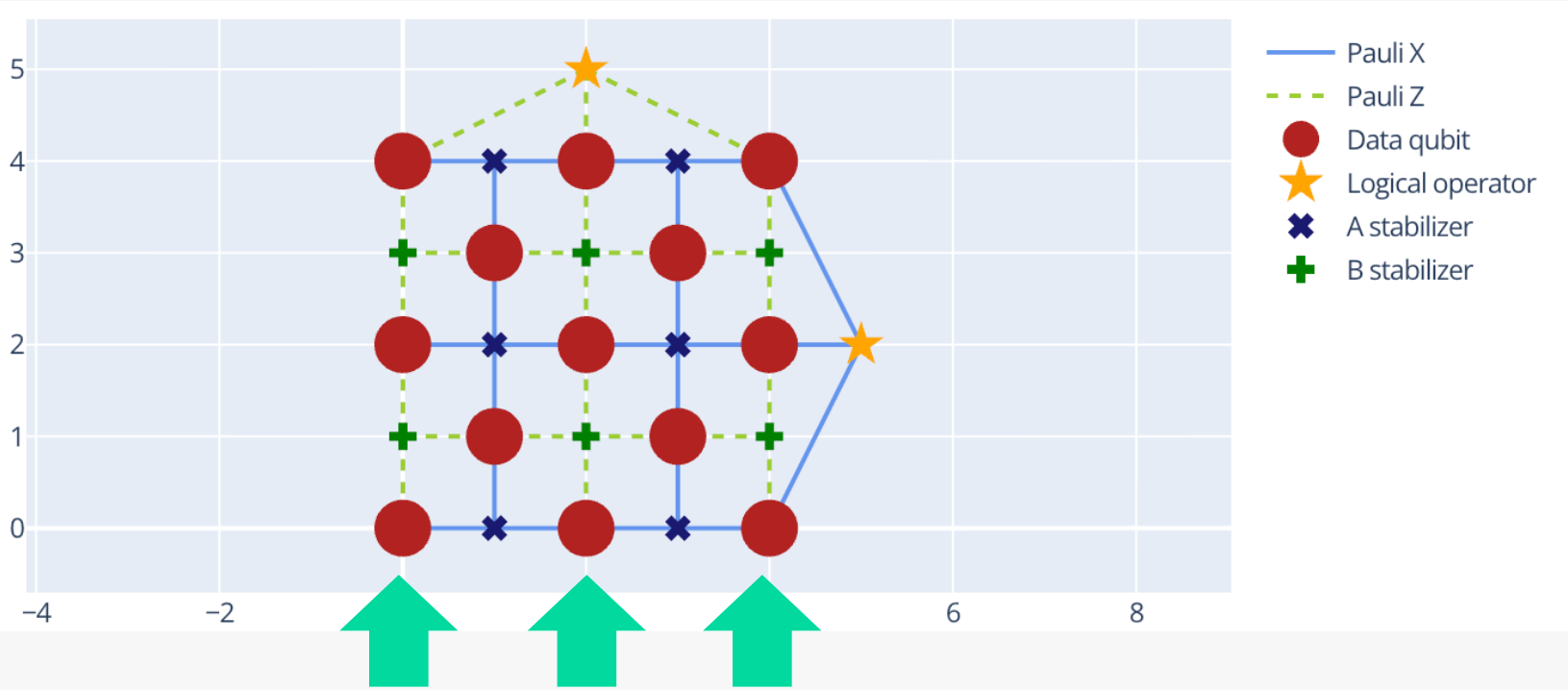
- What stabilizer will be triggered with a X error on below marked qubits?



→ None, it is a logical error (remember it's a distance 3 code)

Planar Code

- What stabilizer will be triggered with a Z error on below marked qubits?



→ None, it is a logical error (remember it's a distance 3 code)

Summary

Summary

- We can use additional qubits to protect our information.
- The stabilizer formalism tells us what kind of non-destructive measurements we have to perform in order to detect errors. We measure multiple syndroms to detect errors
- The smallest number of errors that can result in a logical error is called the distance of the code.
- The quantum repetition code can only detect one type of error. Surface codes can detect any kind of single-qubit errors.